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Dynamic instability in buildings

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ABSTRACT

This paper presents the development of a Single Degree of Freedom (SDOF) model for the calculation of instability thresholds in multistory buildings. The model is restricted to buildings that can be idealized as two dimensional structures but allows consideration of arbitrary mechanism failure modes. Validation of the model is done by computing ground motion scale factors leading to instability and comparing them with results derived from a general multidegree of freedom formulation. The comparisons indicate that the SDOF model provides good predictions of instability, provided the mechanism that controls dynamically is not too disparate from the one used in deriving the model parameters.

INTRODUCTION

The action of vertical forces acting through lateral deformations is known as the P-delta effect. Under static conditions the P-delta effect can be interpreted in terms of equivalent story shears which add to those resulting from the lateral forces (MacGregor and Hage 1977). During seismic response, however, the lateral forces are not prescribed but dependent on the properties of the structure and the P-delta effect is more appropriately interpreted as a reduction in the lateral stiffness.

Although second order effects invariably lead to increases in deformations for static loading, the dynamic response with P-delta effects included may or may not be larger than the first order solution. In particular, studies by Jennings and Husid (1970), Takizawa and Jennings (1975), and Bernal (1990), have shown that gravity typically has a small effect on inelastic response, except when the strength of the structure is near a certain critical value below which the response grows unbounded, indicating failure from instability. Fig.1 illustrates the maximum response versus yield strength for an elastoplastic SDOF system with a first order elastic period of 1.0 second and 5 % of critical damping. Part (a) shows results computed with and without gravity for El Centro and part (b) illustrates the same for Pacoima Dam.

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The small influence of gravity in the inelastic response, except when the strength is close to the instability threshold can be readily observed in both cases.

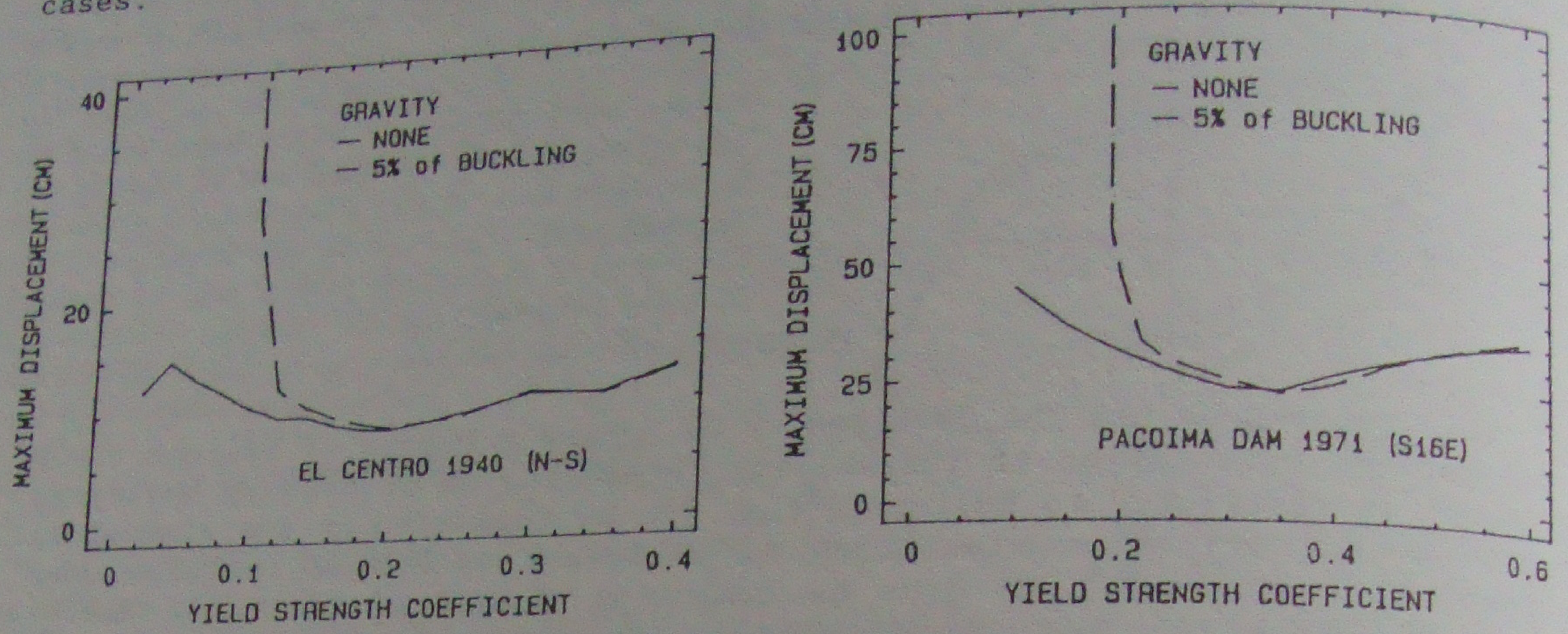

Figure 1. Maximum response of SDOF system (period = 1 sec, 5 % damping)

A conclusion in the preceding discussion is that the importance of the P-delta effect for seismic loading does not stem from amplifications in the inelastic response but derives from the potential for dynamic instability during significant inelastic excursions. The derivation of a SDOF model to asses the strength level associated with the threshold of instability is presented in this paper. Information on instability limit spectral ordinates, as well as additional discussion on the SDOF model, may be found in Bernal (1990).

EQUIVALENT SINGLE DEGREE OF FREEDOM MODEL FOR STABILITY ANALYSIS

We restrict the treatment to buildings that can be modelled as having one horizontal degree of freedom per story. For these class of structures the equations of motion can be written in incremental form as:

$$M\Delta\ddot{u} + C\Delta\dot{u} + (K_t - K_g)\Delta u = - M1\Delta\ddot{x}_h \quad (1)$$

where M and C are the mass and damping matrices, K_t and K_g are the tangent and geometric stiffness matrices, u is the horizontal displacement vector, $\ddot{x}_h$ is the horizontal ground acceleration, and the symbol Δ is used to indicate increment.

The standard approach to obtain a reduction of Eq.1 to one with a SDOF is to assume that the displacement vector is given by the product of a constant shape times an amplitude. It is possible, however, to use a variable shape function, provided the variations are made dependent on the amplitude of the generalized displacement (Biggs 1964). The use of a variable shape conduces to a model that can accommodate arbitrary failure mechanisms and is thus selected here. In particular, the derivation is based on the shapes resulting from a nonlinear static analysis for a preselected lateral load pattern. A brief discussion on the selection of the load pattern is presented later.

Derivation

Consider a structure subjected to a statically applied lateral load of increasing amplitude. Although the load may have an amplitude dependent distribution, we require the shape of the load increments to be constant between yielding events. It follows, accepting the typical idealization of concentrated plastic hinges, that the incremental displacement shape between events is also constant. For any interval of linear behavior between events one can write;

$$K_t \Delta u = p_i L_i \tag{2}$$

and

$$\Delta u = \Delta Y \phi_i \Gamma_i \tag{3}$$

where L_i is the incremental load pattern after the formation of i hinges and p_i is its amplitude; Y is the generalized displacement, ϕ_i is the shape after the formation of i hinges, and Γ_i is a normalizing parameter (chosen for convenience in the derivation as);

$$\Gamma_i = \frac{\phi_i^T M 1}{\phi_i^T M \phi_i} \tag{4}$$

By substituting Eqs.2 to 4 into Eq.1 the equations of motion can be reduced to a SDOF. Performing these substitutions and premultiplying by ϕ_i^T one can write, after some manipulation;

$$\Delta\ddot{Y} + 2\omega_0 \xi_i \Delta\dot{Y} + \Delta S_a - \omega_0^2 \theta_i \Delta Y = -\Delta\ddot{x}_h \tag{5}$$

where

$$\xi_i = \frac{c_i}{2 m_i \omega_0} \tag{6}$$

$$\theta_i = \frac{k_{gi}}{m_i \omega_0^2} \tag{7}$$

$$\Delta S_a = \frac{p_i \phi_i^T L_i}{\Gamma_i \; m_i} \tag{8}$$

and

$$m_i = \phi_i^T \; M \; \phi_i \tag{9}$$

$$c_i = \phi_i^T \; C \; \phi_i \tag{10}$$

$$k_{gi} = \phi_i^T \; K_g \; \phi_i \tag{11}$$

and ω_0 is the elastic (first order) natural circular frequency, given by;

$$\omega_0 = \frac{\phi_0^T K_0 \phi_0}{m_0} \tag{12}$$

In the preceding expressions ξ_i is the damping ratio, θ_i the stability coefficient, S_a the generalized resistance per unit mass and, in Eq.12, K_0 is the elastic stiffness matrix (the subscript 0 is used for elastic conditions).

The computation of the skeleton for the damping, resistance and geometric terms as a function of the generalized displacement Y can be carried out from the results of the monotonically increasing lateral load analysis. These calculations lead to a piece-wise constant damping and stability terms and to a piece-wise linear resistance per unit mass. Although the cyclic behavior of these terms can in principle be derived by tracing the incremental shapes under reversed static loading, this is impractical and unwarranted. Instead, it is appropriate to idealize the skeletons using a small number of parameters and then associate them with a hysteretic rule.

Generalized Resistance per Unit Mass

A bilinear idealization of the generalized resistance can be defined by the initial slope and the maximum value S_{au}; the initial slope is equal to the square of the elastic natural circular frequency. To derive an expression for S_{au} consider the relationship between S_a in the SDOF system and the static base shear in the structure.

The incremental base shear, ΔV, is given by;

$$\Delta V = p_i 1^T L_i \qquad (13)$$

dividing Eq.13 by Eq.8 one gets;

$$\Delta S_a = \frac{\Delta V}{M_{ei}} \qquad (14)$$

where

$$M_{ei} = \frac{1^T L_i \Gamma_i m_i}{\phi_i^T L_i} \qquad (15)$$

Since M_{ei} is not constant, the ultimate resistance per unit mass can only be computed "exactly" by adding increments. It can be shown, however, that little error results if S_{au} is computed as the ratio of the base shear capacity V_u to the elastic value of the effective mass, (Bernal 1990). It is opportune to note that, for buildings with uniform mass distribution, M_{e0} can be taken as 90 % of the total mass without introducing undue error.

Gravity Effect (Stability Coefficient)

The stability coefficient (Eq.7) reflects the influence of gravity in the equation of motion. In the study reported by Bernal (1990), it is shown that this parameter is rather insensitive to the deformation pattern and that only large changes in shape associated with the formation of partial mechanisms lead to significant departures from the elastic value. On this basis it is reasonable to idealize the fluctuations by assuming that θ equals the elastic value for the linear part of the idealized resistance and that it takes the value associated with the mechanism shape in the plastic range.

For buildings having a reasonably uniform distribution of the mass along the height the following formulas apply (Bernal 1990);

$$\theta_0 = \frac{3N\ g r}{(2N+1)\omega_0^2\ h} \tag{16}$$

and

$$\theta_m = \frac{\Omega\ g r}{\omega_0^2\ h} \tag{17}$$

where r is the ratio of total weight to inertial weight (typically calculated at the first story level), g is the acceleration of gravity, h is the total height of the structure, N is the number of stories and Ω is given by;

$$\Omega = \frac{(1 - G/2h - E/h)}{G/h\ (1 - 2G/3h - E/h)} \tag{18}$$

where the parameters E and G depend on the mechanism shape and are defined in Fig.2. In deriving Eq.16 a straight line was used to approximate the elastic shape and Eq.18 was derived by replacing the discrete mass distribution with a continuous one. As expected, the error introduced by the continuum formulation decreases with the number of stories. For one and two story buildings θ_m should be evaluated using Eq.7.

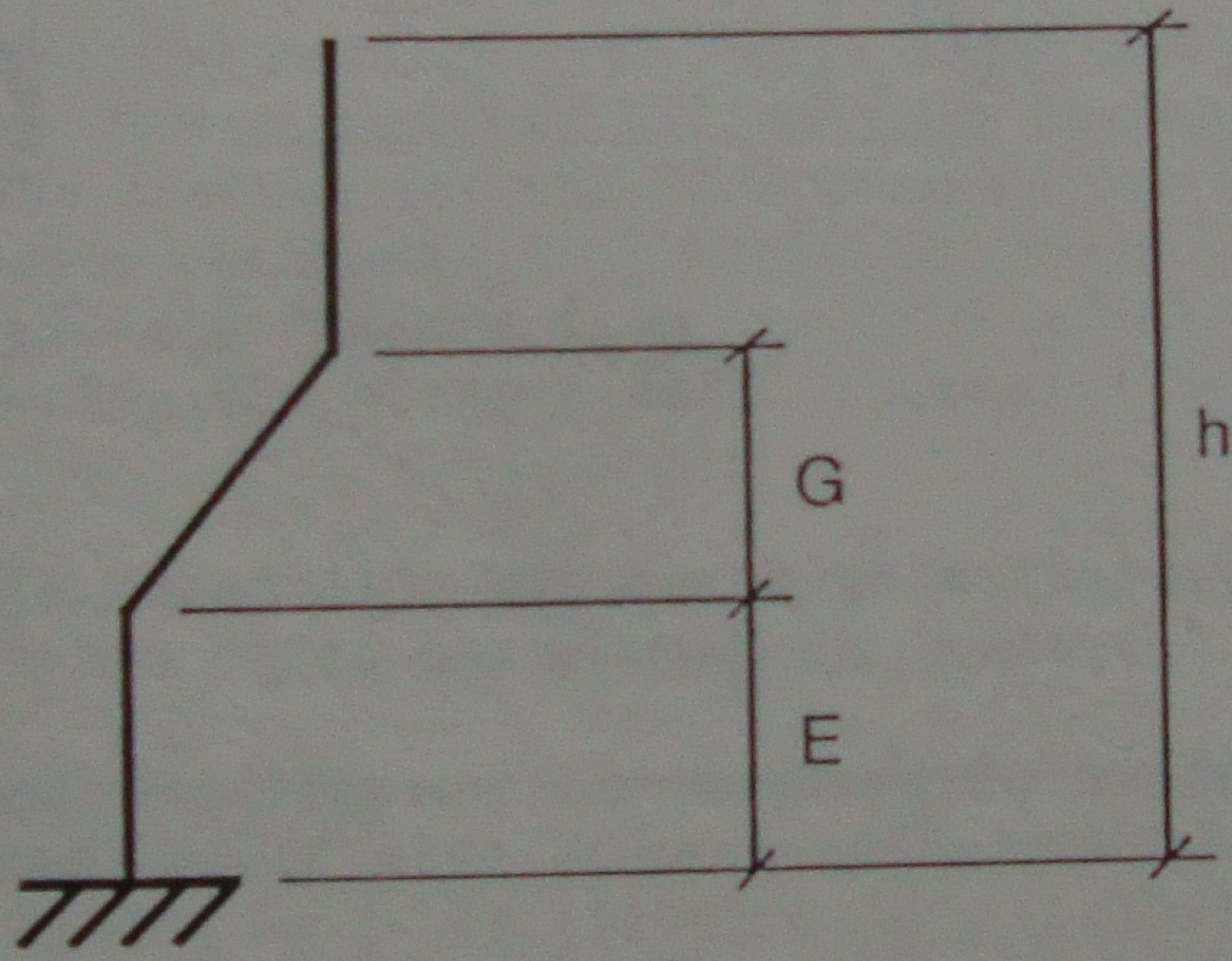

Figure 2. Parameters that define the mechanism shape

It is possible to combine the two stability coefficients into a single equivalent value. Fig.3(a) illustrates a plot of the skeleton of the idealized bilinear effective restoring force with the effect of gravity included in accordance with the two θ idealization previously discussed. In part (b) of the figure, another curve, based on a single stability coefficient, $\bar{\theta}$, is shown. It is apparent by comparing these two plots that they can be made identical by defining equivalent parameters as;

$$\bar{\theta} = \frac{\theta_m}{Q} \tag{19a}$$

$$\bar{\omega}_0^2 = \omega_0^2 \, Q \tag{19b}$$

and

$$\bar{S}_{au} = S_{au} \, Q \tag{19c}$$

where

$$Q = 1 - \theta_0 + \theta_m \tag{19d}$$

by inspection of Eq.5 the equivalent damping is;

$$\bar{\xi} = \frac{\xi}{\sqrt{Q}} \tag{19e}$$

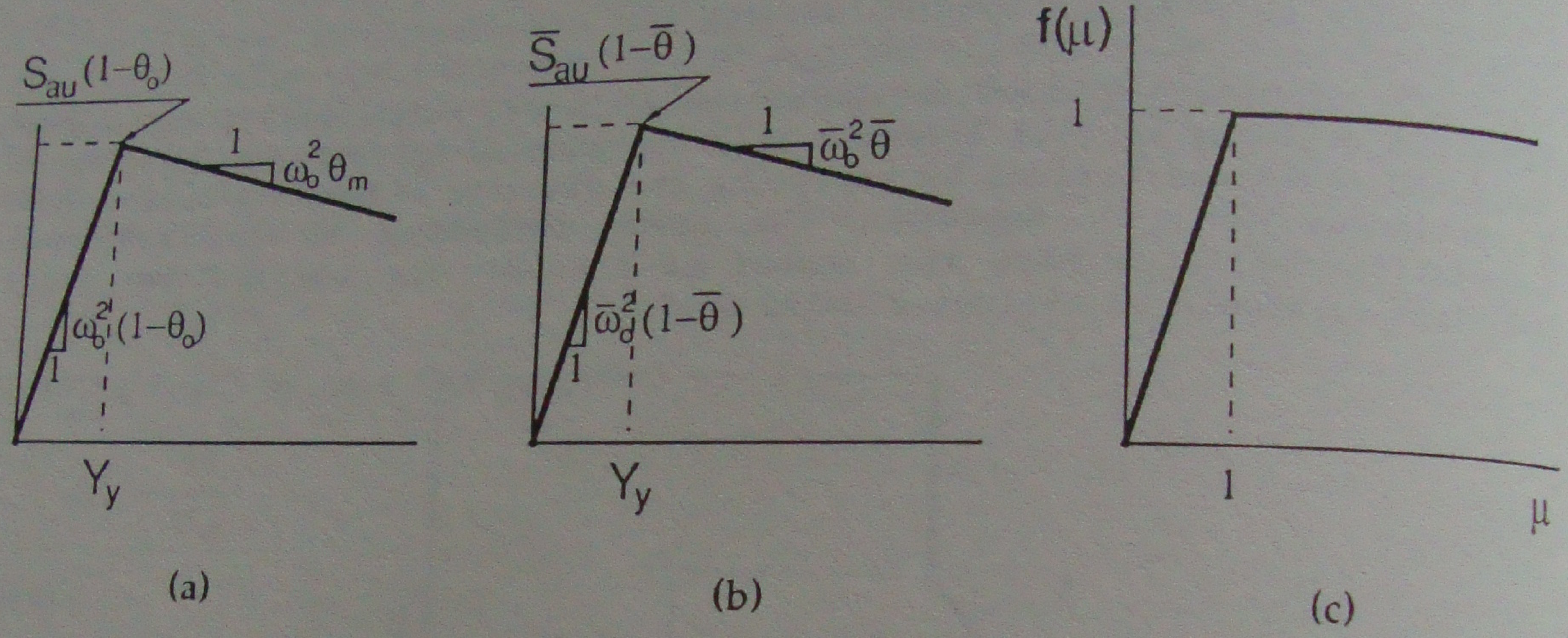

Figure 3. Effective restoring force per unit mass: a) for a two θ model b) equivalent single θ system; c) Normalized first order resistance function.

Using the equivalent parameters and normalizing the equation of motion with respect to the yield displacement Y_y (Fig.3) one can write;

$$\Delta\ddot{\mu} + 2\bar{\omega}_0\bar{\xi}\,\Delta\dot{\mu} + \bar{\omega}_0^{\,2}(\Delta f(\mu) - \bar{\theta}\Delta\mu) = -\frac{\bar{\omega}_0^2\Delta\ddot{x}_h}{\bar{S}_{au}} \tag{20}$$

where

$$\mu = \frac{Y}{Y_y} \tag{21}$$

and $f(\mu)$ is as shown in Fig.3(c)

Eq.20 can be solved numerically to compute the minimum value of $\bar{S}_{au}$ at which the response becomes unstable. It is perhaps appropriate to note that dividing $\bar{S}_{au}$ by a factor is identical to scaling the ground motion and thus the results from Eq.20 can be interpreted in terms of minimum strength for a given record or as a scale factor leading to instability for a fixed value of the available strength $\bar{S}_{au}$.

Selection of the Lateral Load Pattern

It can be shown numerically that, provided there is no change in the form of the controlling kinematic mechanism, the shape of the lateral load has very little influence in the parameters of the equivalent SDOF model. Since most buildings have mechanism shapes that do not vary for a rather wide range of lateral load distributions, the selection of the load pattern is typically not critical. Notwithstanding, research on an approach to define the lateral load pattern, applicable to buildings with sensitive mechanisms, seems warranted.

CORRELATION BETWEEN INSTABILITY PREDICTIONS DERIVED WITH THE EQUIVALENT SDOF SYSTEM AND THOSE OBTAINED USING MDOF MODELS

Comparison of results derived using the equivalent SDOF system and those obtained from MDOF structural representations can be used to asses the accuracy of the simplified model. The earthquake scale factor corresponding to an instability failure is used here for the comparisons. A ten story three bay frame (Anderson and Bertero 1989), a ten story single bay frame (Anderson and Bertero 1969) and a 4 story 3 bay frame (Pique 1976) are chosen for the calculations. Each frame is analyzed using four ground motions. The load pattern chosen to calculate the SDOF parameters is triangular up to the fourth story and uniform for higher levels. This pattern is tentatively recommended for buildings with reasonably uniform mass distribution, on the basis of results from an ongoing study.

Table 1 summarizes the results. The table lists the smallest value of the scale factors at which the structures failed from instability for each one of the ground motions. The factors for the MDOF formulation are shown in the upper part of each box and were obtained using DRAIN-2D (Kannan and Powell 1973). The predicted instability scale factors from the SDOF model (in parenthesis) were computed from Eq.20 using the program INEL1 (Bernal 1990).

Table 1. Comparisons between SDOF and MDOF predictions of scale factors leading to instability

	El Centro (1940) S00E Comp.	I.V.C. (1979) S50W Comp.	Pacoima (1971) S16E Comp.	Taft (1952) S69E Comp.
10 Story 3 bay frame	4.75 (5.12)	1.63 (1.60)	2.25 (2.15)	8.10 (8.10)
10 Story 1 bay frame	4.20 (4.54)	1.50 (1.43)	1.90 (1.88)	8.00 (7.88)
4 Story 3 bay frame	3.10 (3.10)	1.40 (1.55)	1.40 (1.60)	5.60 (6.28)

Values in parenthesis are from the equivalent SDOF model

As can be seen the SDOF predictions are generally in good agreement with the values obtained from the MDOF formulation.

CONCLUSIONS

An equivalent SDOF model to investigate dynamic instability in multistory buildings is presented. The model allows for fluctuations in the dominant shape of the structure and is thus applicable to buildings having arbitrary kinematic mechanisms. Although derived using the sequence of shapes that result from the static application of an increasing lateral load, it is found that the elastic and the mechanism shapes dominate the behavior of the model and, as such, can be used to arrive at a simplified formulation. It is essential, however, that the load pattern chosen lead to a mechanism that approximates the one that actually controls dynamically. Aside from this requirement, the results from the equivalent SDOF model are insensitive to the shape of the loading used. Given a building, definition of the equivalent SDOF requires only the natural period and the shape of the expected failure mode.

Comparisons of SDOF model predictions of instability with results derived using complete MDOF formulations showed good correlation. It should be noted, however, that in the cases tried, the static mechanism provided a reasonable approximation to the manner in which the buildings failed dynamically. Research to establish a general practical approach to predict dynamic failure mechanism modes is needed.

The practicality of the SDOF model presented is fully realized when it is used in conjunction with collapse response spectra to calculate minimum strengths to prevent instability in terms of key ground motion parameters.

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